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STRING MODELS FOR LOCALLY SUPERSYMMETRIC GRAND UNIFICATION*

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ABSTRACT

Phenomenologically viable string vacua may require incorporating Kac-Moody algebras at level ≥ 2. We exploit the free fermionic formulation to construct N=(0,2) world-sheet supersymmetric string models with specific phenomenological input: N=1 spacetime supersymmetry, three generations of chiral fermions in gauge groups $SO(10)$ or $SU(5)$, adjoint Higgses, and a single Yukawa coupling of a fundamental Higgs to the third generation. In this talk, we will show models of gauge group $SO(10)$ and of $SU(5)$ without any gauge singlet moduli, and show some novel features appearing in the connection of these two models. The accompanying, and rather non-trivial, discrete chiral sub-algebras can determine hierarchies in the fermion mass matrix. Our approach to string phenomenology opens up the possibility of *concrete* explorations of a wide range of proposals both for dynamical supersymmetry breaking and for the dynamics of the dilaton and other stringy moduli.

Model building in string perturbation is not fully explored, partly because there exists millions of vacua (and each vacuum, if “realistic”, is cumbersome to analyze in full details), and partly because we have not really tried hard enough to explore this area in an intelligent way. For instance, there are only three basic examples of superstring GUT constructions in the literature[1–3] in spite of thousand papers on traditional GUT models.

We have chosen to base our exploration of string vacua on four dimensional closed free fermionic string models, which are heterotic superstring vacua described by a world–sheet lagrangian for 64 real (Majorana-Weyl) free fermions, together with two

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bosons representing the two transverse coordinates of 4-d spacetime in the light-cone formalism. This construction is described in detail in refs[4–7]; we will, for the most part, follow the notation and conventions of refs.[7,1]

Models are conveniently specified by their one-loop partition functions, which include all the spacetime particle spectrum; these involve a sum over spin structures:

$$Z_{\rm fermion} = \sum_{\alpha,\beta} C^\alpha_\beta \, Z^\alpha_\beta \quad , \tag{1}$$

where the C^α_β's are numerical coefficients, while α and β are 64-dimensional vectors labeling different choices of boundary conditions for the fermions around the two independent cycles of the worldsheet torus. For each real fermion there are two possible choices of boundary conditions around a given cycle: either periodic (Ramond) or antiperiodic (Neveu-Schwarz). However for fixed α and β the real fermions always pair up into either Majorana or Weyl fermions; if a particular Weyl pairing occurs consistently across *all* α and β, then this pair can be regarded as a single *complex* fermion. For such complex fermions more general boundary conditions – any rational "twists" – are then allowed.[4,6,8] A useful notation denotes a complex Ramond fermion as a $-1/2$ twist, while a general m/n twist indicates the complex fermion boundary condition

$$\psi \to \exp\left[2\pi i \,\frac{m}{n}\right] \,\psi \quad . \tag{2}$$

The contribution of any sector α to the partition function contains a generalized GSO projection operator. Up to an overall constant, this is given by:

$$\sum_\beta \, C^\alpha_\beta \, \exp\left[-2\pi i \beta\cdot \hat N(\alpha)\right] \quad , \tag{3}$$

where $\hat N(\alpha)$ is the fermion number operator defined in the sector α. There are subtleties in the proper definition of $\hat N(\alpha)$ for real Ramond fermions; these are discussed in ref. [7]. Thus building a fermionic string model amounts to choosing an appropriate set of α's, β's, and C^α_β's, then performing the GSO projections to find the physical spectrum. These choices are greatly constrained by the requirement of modular invariance of the one-loop partition function; in addition, higher loop modular invariance imposes a factorization condition on the C^α_β's. Together these requirements imply that the $\{\beta\}$ are the same set of vectors as the $\{\alpha\}$, and that, if two sectors α_1 and α_2 appear in the partition function, then the sector $\alpha_1+\alpha_2$ must also appear. These facts allow one to specify the full set of α's and β's by a list of "basis vectors", denoted V_i, and we will later express α in terms of these basis vectors with coefficients α_i. The C^α_β's parametrizing the generalized GSO projection operators in eq. (3) can also be reexpressed in the same basis in terms of new parameters k_{ij}; these are discussed in [4].

Models that consist of real fermions can have higher level Kac–Moody algebra and gauge group of rank lower than 22, which is an advantage if one wants to build up a realistic model. However, the spacetime particle spectrum in such models are generally realized in rather intricate fashion, and quite different from level one models. For example, the 45 gauge bosons of $SO(10)$ at level 2 in the model we will later present reside in both the untwisted sector and 7 other twisted sectors. Furthermore, there are probably 200 twisted sectors containing massless spectrum before GSO projection in a typical model of three generations of chiral fermions $\mathbf{16}_L$. Fortunately, these complications and tedious checks can be easily handled within seconds by our newly developed symbolic manipulation computer package. The details regarding this package will be explained in our later publication.[9]

Equipped with this powerful tool, one can start to explore the string vacua which incorporate a considerable amount of low–energy phenomenology input. Studies along this direction can help us to extract some interesting features about true string vacua which are closer to nature, and it may also reveal hints for eventual non-perturbative formulation of string theory. To give one example besides models with three generations of chiral fermions and adjoint scalars,[3] we will present a model with N=1 spacetime SUSY, three generations of chiral fermions in gauge group $SO(10)$ at level two, and *no moduli* (except dilaton). Moreover, we will show, from this $SO(10)$ model, one can obtain a model of $SU(5)$ at level two with three chiral 10s by just tuning parameters k_{ij} in the GSO projection operators.

The first model has basis vectors V_0 – V_9 specified as follows:

$$
\begin{aligned}
V_0 &= (11111111111111111111\|111111111111|11111111111111|111111111111111111)\\
V_1 &= (11100100100100100100\|000000000000|00000000000000|000000000000000000)\\
V_2 &= (00000000000000000000\|111111110000|11111111000000|000000000000000000)\\
V_3 &= (00000000000000000000\|000000000000|00001111111100|000000000000000000)\\
V_4 &= (00000000000000000000\|110000111111|11001100110011|000000000000000000)\\
V_5 &= (11100100010010010010\|111100001100|10101010101010|111000000000000000)\\
V_6 &= (11010010100100001001\|111100001100|10100101101001|000001110000000000)\\
V_7 &= (11001001001001100100\|111100001100|11110000111100|000000001100000000)\\
V_8 &= (001011011101100{\scriptstyle +-}0{\scriptstyle +-}\|00000000{\scriptstyle ++++}|01010101010101|000111{\scriptstyle ++++++--}0000)\\
V_9 &= (00{\scriptstyle +-}0{\scriptstyle +-}0{\scriptstyle ++}1{\scriptstyle ++}1000000\|000000000000|00001111000011|0011001100{\scriptstyle ++++++++})
\end{aligned}
$$

"1" in the above vectors denote the value "$-1/2$", and "$\pm$" denote the values of "$\pm 1/4$". The first 20 components up to the double vertical lines denote the boundary conditions for the right-moving fermions. The first pair of right-movers are spacetime fermions (corresponding to the superpartners of the two transverse directions in 4-d), while the other 18 right-movers are "internal". The remaining 44 components are left-movers. The first 12 components on the right of the double vertical lines denote the quantum numbers under $SO(10) \times U(1)$, and the second 14 components are real fermions necessary for this particular $SO(10)$ embedding.

In fermionic string models, there exists an "untwisted sector", with 64 Neveu-Schwarz Weyl fermions. The untwisted sector contains the graviton, dilaton, antisymmetric tensor field, and some of the gauge bosons. The requirement of a worldsheet supercurrent constructed out of the right-movers and the spacetime bosons is a consistency constraint on model building. As a result, V_1 contains massless gravitinos, corresponding to an N=4 spacetime supersymmetry before the GSO projection. After the projection one only has N=1 spacetime SUSY. Superpartners of states in some sector α will be found in $V_1+\alpha$. V_0 is required in all fermionic string models by modular invariance. To produce a model with $SO(10)\times U(1)$ and three generations of chiral fermions, we also have to choose k_{ij} with $i>j>0$ or $i=j=0$,

$$k_{00}=1/2,\quad k_{61}=1/2,\quad k_{73}=1/2,\quad k_{83}=1/2,\quad k_{86}=1/2,\quad \text{and the rest}=0. \tag{4}$$

The 45 gauge bosons of $SO(10)$ at level 2 reside in the untwisted sector, V_2, V_3, V_4, V_2+V_3, V_2+V_4, V_3+V_4, and $V_2+V_3+V_4$. One can check that the root vectors forming from the "fermionic charges" associated with the first 12 left-moving fermions, or first six complex fermions, do have length 1; thus it is a level 2 Kac–Moody algebra. Three generations of 16_L are contained in $\{V_5,V_6,V_7\}+\{0,V_2,V_4,V_2+V_3,V_2+V_4,V_3+V_4,V_2+V_3+V_4\}$. Notice that $V_{5,6,7}+V_3$ are projected out. In this model, the observable gauge group is $SO(10)\times U(1)^4$, and the hidden gauge group is $U(1)^3\times U(2)$. There are no moduli in this model, *i.e.*, no gauge singlet with respect to both observable and hidden sectors. However, we do have 8 states, half of them contained in V_3 and the other half in V_3+2*V_9, which are gauge singlets with respect to the observable gauge group but not gauge singlets under the hidden gauge group. It is worthwhile to point out that in this construction one uses 26 fermions with central charge $c=13$ to construct a conformal field theory based on Kac–Moody algebra of $SO(10)$ at level 2 tensoring $U(1)$, whose total central charge is $c=10$. The precise form of the remaining chiral algebra could be worked out.

Now if one chooses the same set of k_{ij} values except changing $k_{93}=1/2$, one would obtain a model with observable gauge group $SU(5)\times U(1)^5$, while the hidden gauge group is unchanged. The gauge bosons associated with $V_3,V_2+V_3,V_3+V_4,V_2+V_3+V_4$ do not pass the GSO projection. The remaining gauge bosons form exactly the root lattice of $SU(5)$, and the level is still 2 because the root length is unchanged. It is interesting to point out that the original 16_L's of $SO(10)$ from $V_{5,6}$ and their gauge partners become $10_L+\bar{5}_L+1_L$ of $SU(5)$, while the 16_L from V_7 become $10_L+\bar{5}_R+1_R$, and an additional $\bar{5}_L+1_L$ appears in $V_7+V_9+\{V_2+V_3,V_3+V_4,V_2+V_3+V_4\}$.

Our experience in model building tells us that it is not difficult to get models with three generations of chiral fermions with adjoint Higgs, or with no moduli, or with only one Yukawa coupling to the third generation. But it may take some effort to construct a model with all the desired features.

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